Syllabus for Ph.D. Entrance Exam

Physics

1. MATHEMATICAL PHYSICS

Linear algebra: Vector spaces, basis and dimensions, linear operators, matrices, eigenvalues and eigenvectors, inner product, orthogonality, self-adjoint and unitary transformations, diagonalization ***Complex analysis:*** Analytic functions, Cauchy-Riemann conditions, Cauchy's theorem, Taylor and Laurent series, calculus of residues, contour integrations, introduction to analytic continuation and Riemann surfaces, **Fourier analysis:** Fourier series, Fourier integral and transform, convolution theorem, Parseval's identity, applications to solving differential equations, Laplace transform and applications to differential equations, ***Special functions:*** Legendre, Hermite functions, generating function, recurrence relations, Bessel's function of 1 st kind, spherical Bessel function, spherical harmonics.

2. CLASSICAL MECHANICS

Lagrangian formulation: Mechanics of a particle, Mechanics of a system of particles, Constraints, Generalized co-ordinates, D'Alembert's principle, Lagrange's equations of motion, Simple applications of the Lagrangian formulation, Galilean invariance of Lagrange's equations, ***Variational principle:*** Hamilton's principle; Some techniques of the calculus of variations – applications – shortest distance problem, Brachistochrone; Derivation of Lagrange's equation from Hamilton's principle; Conservation theorems and symmetry properties – integrals of motion, cyclic co-ordinates, Jacobi's integral, ***Central force:*** Two body central force problem – Reduction to the equivalent one body problem; Equations of motion and first integrals; Classification of orbits; The Virial theorem; Differential equation for the orbit, integrable power-law potentials; The Kepler problem – inverse square law of force, motion in time in Kepler problem, ***Hamiltonian formulation:*** Legendre transformations, Hamilton's equations of motion – Canonical variables; Cyclic co-ordinates and conservation theorems in Hamiltonian formulation; Derivation of Hamilton's equations from a variational principle; Canonical transformations – Generating functions, examples; Poisson brackets and other canonical invariants; Equations of motion and conservation theorems in Poisson bracket formulation; Phase-space; Liouville's theorem ***Continuum Mechanics:*** Strings, D'Alembert's solution to the wave equation – Energy density and energy current; Reflection at an interface; Mass point on a string; Interface between strings of different mass density, Finite strings – Bernoulli's solution, Sturm-Liouville Theory – Variational method, Continua in Higher dimensions –

Membranes; Helmholtz equation; Rectangles; Circles; Sound in fluids, Dispersion, ***Rotational motion:*** Rotating frame of reference, inertial forces in rotating frames, Coriolis force, Foucault pendulum, Larmor's precession

3. QUANTUM MECHANICS:

Motivation for need for Quantum Mechanics: Inadequacy of classical Physics, wave packets and uncertainty relations. Schrodinger wave equation and probability interpretation, Concepts of probability (with emphasis on problem solving), ***One Dimensional Problems:*** Particle in an infinite potential as a prototypical problem in quantum mechanics: Wavefunctions, Energy Eigenvalues, Superposition of Wavefunctions (comparison with Fourier Series), Free particle wavefunction - comparison with the infinite potential, Momentum Wavefunction (Fourier transform and Parseval's Theorem - normalization in position space and momentum space; use of Contour Integrals). Momentum and position expectation values in Momentum space Finite wells and barriers Simple Harmonic Oscillator: Analytical Method, ***Postulates of quantum mechanics:*** Representation of states and dynamical variables, observables, self-adjoint operators, eigenfunctions and eigenvalues, degeneracy, Dirac delta function, Completeness and closure property, Physical interpretation of eigenvalues, eigenfunctions and expansion coefficients, eigenvalues and eigenfunctions of momentum operator. Hilbert space, Dirac's bra and ket notation, dynamical variables and linear operators, projection operators, unit operator, unitary operator, matrix representation of an operator, change of basis, unitary transformation. Eigenvalues and eigenfunctions of simple harmonic oscillator by operator method. Ehrenfest Theorem, ***Time Evolution of system*:** Constants of motion, Schrodinger and Heisenberg picture, ***Quantum Mechanics in Two Dimensions*:** with Particle in a 2-D box as example. Degeneracies. ***Quantum Mechanics in Three Dimensions:*** Separation of variables and orthogonalization; ***Hydrogen Atom Problem*:** Radial Solutions.

4. ELECTRODYNAMICS

Electric fields and Potentials: Gauss's Law and its application, Electric potential, Poisson's and Laplace's equation, Boundary value problems (Method of images). ***Multipole expansions and material media:*** Multipole expansions for a localised charge, distribution in free space, Magnetostatics- magnetic vector potential, it's multipole. expansion, static electric and magnetic fields in material media, Boundary conditions, **Time varying fields:** Time dependents field, Faraday's law for stationary and moving, media, Maxwell's displacement current, Differential and Integral forms of Maxwell's equations, Maxwell's equations for material medium. ***Energy, Force and Momentum relations in electromagnetic fields:*** Energy relations in quasi-stationary current systems, Magnetic interaction between two current loops, Energy stored in electric and magnetic fields, Poynting's theorem, General expression for electromagnetic energy, Conservation laws. ***Electromagnetic wave equations:*** Electromagnetic wave equations, Electromagnetic plane waves in stationary medium, Reflection and refraction of electromagnetic waves at plane boundaries (Oblique incidence), Electromagnetic waves in conducting medium, Skin effect and skin depth. ***Inhomogeneous wave equations:*** Inhomogeneous wave equations, Lorentz's and Coulomb's gauges, Gauge transformations, Wave equations in terms of electromagnetic potentials, D'Alembertian operator, Dipole radiation, Radiation energy and Radiation resistance. ***Relativistic Kinematics:*** Experimental basis for special theory of relativity (Michelson - Morley experiment), 2 Lorentz transformations, Relativistic velocity addition, Mass- Energy relation ($E=mc^2$). ***Covariance and Relativistic Mechanics:*** Minkowski's space-time diagram, light cone, Four vectors, Lorentz transformation of Four vectors, Some tensor relations useful in special relativity, Minkowski's force. ***Covariant formulation of electrodynamics:*** Magnetism as relativistic phenomenon, Electromagnetic field tensor, Lorentz force on a charged particle.

5. ATOMIC AND MOLECULAR PHYSICS

Atoms: Electromagnetic spectrum, Types of molecular energies, Hyperfine structure, Width of spectral line, Nuclear spin, Normal and anomalous Zeeman effect, Paschen - Back effect. Lamb Shift, Schrodinger Wave equation for two electron system many electron atoms: LS and JJ coupling schemes, Lande interval rule. ***Molecules:*** Rotational and Vibrational spectra for diatomic molecules, Electronic spectra of diatomic Molecule, Vibrational coarse structure, Vibrational analysis of band system, Frank-Condon principle, Dissociation energy & dissociation products, Rotational fine structure of electronic vibration transitions, Electronic angular momentum in diatomic molecule, NMR Spectroscopy : Nuclear spin magnetic moment, Interaction of nuclear magnet with external magnetic field, NMR spectrometer, chemical shift,spin spin coupling splitting of NMR signals, Applications. ***ESR* Spectroscopy:** Electron spin interaction with external magnetic field, Simple ESR Spectrometer, ESR spectrum, Applications, ***Solids:*** Laue theory of X-ray diffraction, Geometrical structure factor, Atomic scattering factor, calculations for bcc, fcc & diamond structure. ***Nuclear Quadrupole Resonance:*** Electric field gradient, principle of NQR, transitions for axially symmetric and non-symmetric systems, NQR instrumentation, Applications.

6. STATISTICAL PHYSICS AND THERMODYNAMICS

Statistical Description of System of Particles: Specification of the state of the system, Macroscopic and Microscopic states, Phase space, Statistical ensemble, Postulate of equal a priori probability, Probability calculations, Behaviour of density of states, Liouville's theorem (Classical), Quasi-static processes, ***Statistical Thermodynamics :*** Equilibrium conditions and constraints, Distribution of energy between systems in equilibrium, Approach to thermal equilibrium, Temperature, Heat reservoir, Sharpness of the probability distribution, Dependence of the density of states on the external parameters, Equilibrium between interacting systems, ***Classical* Statistical Mechanics :** Microcanonical ensemble, System in contact with heat reservoir, Canonical ensemble, Applications of canonical ensembles (Paramagnetism, Molecule in an ideal gas, Law of atmosphere), System with specified mean energy, Calculation of mean values and fluctuations in a canonical ensemble, Connection with thermodynamics, Grand-canonical ensemble, Physical interpretation of a, Chemical potential in the equilibrium state, Mean values and fluctuations in grand canonical ensemble, Thermodynamic functions in terms of the Grand partition function., ***Applications of Statistical Mechanics :*** Classical partition functions and their properties, Calculations of thermodynamic quantities, Ideal mono-atomic gas, Gibbs paradox, Equipartition theorem and its Simple applications. i) Mean kinetic energy of a molecule in a gas ii) Brownian motion iii) Harmonic Oscillator iv) Specific heat of solid. Maxwell velocity distribution, Related distributions and mean values,

Quantum Statistics of Ideal Gases: Symmetry of wave functions, Quantum distribution functions, Boltzmann limit of Boson and Fermion gases, Evaluation of the partition function, Partition function for diatomic molecules, Equation of state for an ideal gas, The quantum mechanical paramagnetic susceptibility, ***Ideal Bose System:*** Photon gas - i) Radiation pressure ii) Radiation density iii) Emissivity iv) Equilibrium number of photons in the cavity. Einstein derivation of Planck's law, Bose-Einstein Condensation, Specific heat, Photon gas - Einstein and Debye's model of solids. ***Ideal Fermi System:*** Fermi energy, Mean energy of fermions at absolute zero, Fermi energy as a function of temperature, Electronic specific heat, White - Dwarfs, Compressibility of Fermi gas, Pouli's paramagnetic, A relativistic degenerate electron gas.

7. SOLID STATE PHYSICS

Crystal physics: Symmetry operations; Bravais lattices; Point and space groups; Miller indices and reciprocal lattice; Structure determination; diffraction; X-ray, electron and neutron; Crystal binding; Defects in crystals; Point and line defects. ***Lattice vibration and thermal properties:*** Einstein and Debye models; continuous solid; linear lattice; acoustic and optical modes; dispersion relation; attenuation; density of states; phonons and quantization; Brillouin zones; thermal conductivity of metals and insulators. ***Electronic properties:*** Free electron theory of metals; electrons in a periodic potential; Bloch equation; Kronig-Penny model; band theory; metal, semiconductor and insulators; band-gap, intrinsic and extrinsic semiconductors, Hall Effect, p-n junction. ***Dielectrics:*** Polarizability; Clausius-Mossotti formula; Dielectric constant; ferroelectrics. ***Magnetism:*** Diamagnetism, paramagnetism, ferromagnetism, antiferromagnetism and ferrimagnetism. ***Superconductivity:*** Meissner effect; London equations; coherence length; type-I and type II superconductors.

<u>Blueprint for Syllabus</u>

	Topics	**Marks Distribution**
1.	Mathematical Physics	15%
2.	Classical Mechanics	15%
3.	Quantum Mechanics	15%
4.	Statistical Physics and Thermodynamics	15%
5.	Electrodynamics	15%
6.	Solid State Physics	10%
7.	Atomic Physics	15%
		100 %